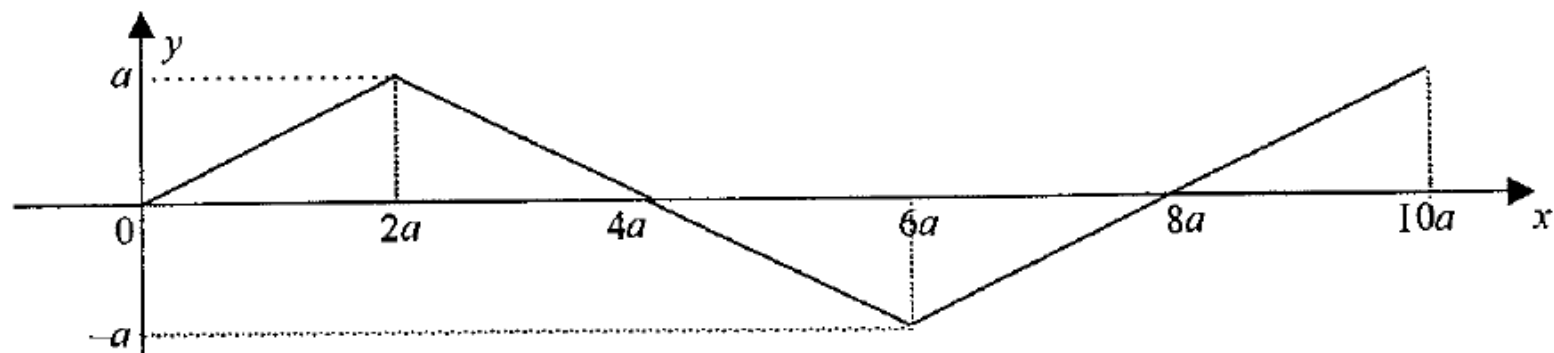


CORE MATHEMATICS (C) UNIT 1 TEST PAPER 6

1. The equations of two straight lines are $2x = y + 3$ and $2y = x + 3$.
 - (i) Determine by calculation whether the lines are perpendicular. [2]
 - (ii) Calculate the coordinates of the point where the lines intersect. [2]

2. (i) Express $(1 + \sqrt{2})^2$ in the form $a + b\sqrt{2}$, where a and b are integers to be found. [2]
 - (ii) By substituting $x = y^{\frac{1}{2}}$, or otherwise, solve the equation $6y^{\frac{1}{2}} = y + 9$. [4]

3. The diagram shows the graph of $y = f(x)$.



Sketch, for $0 \leq x \leq 8a$, graphs of (i) $y = f(x + a)$, (ii) $y = f(x) + a$.

In each case, indicate clearly the coordinates of any points of intersection with the axes. [6]

4. Solve for x the equations

(i) $5^{2x+1} = \frac{1}{25}$,

(ii) $\sqrt{x+3} = x-3$.

[7]

5. The circle C has equation $x^2 + y^2 - 2x + 5y + 7 = 0$.

(i) Find the coordinates of the centre of C and show that the radius of C is $\frac{1}{2}$.

[4]

(ii) Show that the point $A(1, -2)$ lies on C and find the coordinates of the point B such that AB is a diameter of C .

[3]

6. Given that $W = 18v + \frac{2500}{v}$, where $v > 0$,

(i) find $\frac{dW}{dv}$ and $\frac{d^2W}{dv^2}$ in terms of v .

[4]

(ii) Find the value of v when $\frac{dW}{dv} = 2$.

[3]

CORE MATHEMATICS 1 (C) TEST PAPER 6 Page 2

7. In this question, $f(x) \equiv 1 - 5x$ and $g(x) \equiv \frac{16}{1 - 5x}$.

Solve for x the equations

(i) $f(x) = g(x)$, [4]

(ii) $f\left(\frac{1}{x}\right) = g(x)$. [6]

8. The point $(5, k)$ lies on the curve $y = x(8 - x)$.

(i) Find the value of k . [1]

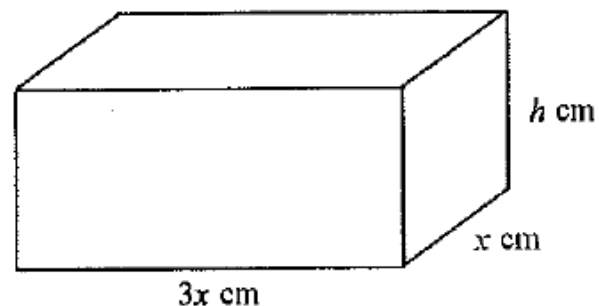
The tangent to the curve $y = x(8 - x)$ at $(5, k)$ meets the x -axis at P . This tangent also meets the line of symmetry of the curve at Q .

(ii) Find the coordinates of P and Q . [7]

R is the point where the curve cuts the x -axis, for $x > 0$.

(iii) Find the area of the triangle PQR . [3]

9. A closed rectangular box has width x cm, length $3x$ cm and height h cm, as shown.



- (i) Show that the total surface area of the box is $2x(3x + 4h)$ cm². [4]
- (ii) Find an expression in terms of x and h for the volume of the box. [2]
- (iii) Given that the surface area has a fixed value of 450 cm², show that $h = \frac{225 - 3x^2}{4x}$. [2]
- (iv) Use differentiation to find the value of x for which the volume is maximum subject to the surface area being 450 cm², and state this maximum volume. [6]

CORE MATHS 1 (C) TEST PAPER 6 : ANSWERS AND MARK SCHEME

- | | | | |
|----|---|-------------|---|
| 1. | (i) Gradients are 2 and $\frac{1}{2}$, so not perpendicular | M1 A1 | |
| | (ii) $4y - 6 = y + 3$ $y = 3$ Point is (3, 3) | M1 A1 | 4 |
| 2. | (i) $1 + 2 + 2\sqrt{2}$ $a = 3, b = 2$ | M1 A1 | |
| | (ii) $6x = x^2 + 9$ $x^2 - 6x + 9 = 0$ $x = 3$ $y = 9$ | M1 A1 M1 A1 | 6 |
| 3. | (i) Translated a units to the left, through $(-a, 0), (0, a/2), (3a, 0), (7a, 0)$ | B3 | |
| | (ii) Translated a units upwards, through $(0, a), (6a, 0)$ | B3 | 6 |
| 4. | (i) $2x + 1 = -2$ $x = -3/2$ | M1 A1 | |
| | (ii) $x + 3 = x^2 - 6x + 9$ $x^2 - 7x + 6 = 0$ $(x - 1)(x - 6) = 0$ | M1 A1 A1 | |
| | $x = 1$ or 6 Must have $x - 3 > 0$, so $x = 6$ | A1 A1 | 7 |
| 5. | (i) $(x - 1)^2 + (y + 5/2)^2 - 1/4 = 0$ Centre = $(1, -5/2)$, radius = $1/2$ | M1 M1 A1 A1 | |
| | (ii) $1 + 4 - 2 - 10 + 7 = 0$ $B = (1, -3)$ | B1 M1 A1 | 7 |
| 6. | (i) $\frac{dW}{dv} = 18 - \frac{2500}{v^2}, \frac{d^2W}{dv^2} = \frac{5000}{v^3}$ | M1 A1 M1 A1 | |

	(ii) $18 - \frac{2500}{v^2} = 2$	$v^2 = 2500/16$	$v > 0$ so $v = 50/4 = 12.5$	M1 A1 A1	7
7.	(i) $(1 - 5x)^2 = 16$	$1 - 5x = 4$ or -4	$x = -3/5$ or $x = 1$	B1 M1 A1 A1	
	(ii) $1 - \frac{5}{x} = \frac{16}{1 - 5x}$	$x(1 - 5x) - 5(1 - 5x) = 16x$		M1 A1 A1	
	$5x^2 - 10x + 5 = 0$	$5(x - 1)^2 = 0$	$x = 1$	M1 A1 A1	10
8.	(i) $k = 5(3) = 15$			B1	
	(ii) Gradient = $8 - 2x = -2$ at $(5, 15)$	Tangent is $y - 15 = -2(x - 5)$		M1 A1 M1 A1	
	P is $(12.5, 0)$	At Q , $x = 4$ so Q is $(4, 17)$		A1 M1 A1	
	(iii) R is $(8, 0)$	Area = $9/4 \times 17 = 153/4$		B1 M1 A1	11
9.	(i) $A = 2(xh) + 2(3xh) + 2(3x^2) = 6x^2 + 8xh = 2x(3x + 4h)$			M1 A1 M1 A1	
	(ii) $V = 3x^2h \text{ cm}^3$			M1 A1	
	(iii) If $A = 450$, $3x^2 + 4xh = 225$	$h = (225 - 3x^2)/4x$		M1 A1	
	(iv) Then $V = 675x/4 - 9x^3/4$	$dV/dx = 675/4 - 27x^2/4 = 0$ when $x = 5$		M1 A1 M1 A1	
	$h = 15/2$	$V_{\max} = 562.5$		M1 A1	14